Economic Growth Models

Harrod – Domar Growth Model
Solow Growth Model
Endogenous Growth Model
Harrod-Domar Model

- \( s/\theta \approx g^* + n + \delta \)

- Per capita growth rate \( g^* \) is a function of saving rate \( s \)
- \( n \) is the population growth rate
- Depreciation rate of capital \( \delta \)
- Capital-output ratio is \( \theta \)
Robert Solow (b. 1924)

- Got the Nobel prize in economics 1987

- Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel

- Added labor and technology to the H-D model
Wit & Wisdom of Bob Solow

- Everything reminds Milton Friedman of the money supply. Everything reminds me of sex, but I try to keep it out of my papers.

- Over the long term, places with strong, distinctive identities are more likely to prosper than places without them. Every place must identify its strongest most distinctive features and develop them or run the risk of being all things to all persons and nothing special to any...Livability is not a middle-class luxury. It is an economic imperative."

- "There is no evidence that God ever intended the United States of America to have a higher per capita income than the rest of the world for eternity."\(^{1}\)
Extension to the Harrod-Domar model

- Solow extended the Harrod-Domar model by
  - Adding labor as a factor of production
  - Requiring diminishing returns to labor and capital separately, and constant returns to scale for both factors combined
  - Introducing a time-varying technology variable distinct from capital and labor
  - The capital-output and capital-labor ratios are not fixed as they are in the Harrod-Domar model.
What it says

- Economies will conditionally converge to the same level of income if they have the same rates of savings, depreciation, labor force growth and development.

- It provides the basic framework for the study of convergence across countries.
Production function

- Relates production (output, or income) to factors (or inputs)
- \( Y = F(K, L) \)
- Graph a production function with diminishing returns to \( K \)
- Cobb-Douglas Production function
  \[ Y = AK^{(1-\alpha)}L^{\alpha} \]
  - Each factor has diminishing returns
  - Both combined are constant returns to scale
Constant Returns to Scale

- \( Y = F(K, L) \)
- If you multiply each factor by \( \lambda \), then the output also goes up by \( \lambda \)
- Do this for the Cobb-Douglas production function
Output as a function of capital

- Per capita output $y$ and per capita capital $k$
- $Y = F(K,L)$
- Divide both sides by $L$
- $Y/L = F(K/L, 1)$
- $y = f(k)$
- Plot output $y$ as a function of $k$, noting that there’s diminishing returns to capital
Solow Growth Model

- Capital stock grows every period
  - \( K(t+1) = (1 - \delta)K(t) + S(t) \)

- If population grows at rate \( n \), then per capita capital stock \( k \) grows at?
  - Note \( P(t + 1) = (1 + n)P(t) \)
  - \( k(t) = K(t)/P(t) \) and \( k(t + 1) = K(t + 1)/P(t + 1) \)

- Rearranging: \( (1 + n)k(t + 1) = (1 - \delta)k(t) + sy(t) \)
  - RHS is depreciated per capita capital and current per capita savings
  - LHS is new per capita capital – modified by the population growth rate
Alternate Form

• $\Delta k = sf(k) - (\delta + n)k$

• Capital-labor ratio $k = K/L$

• Shows that the growth of $k$ depends on the savings $sf(k)$, after allowing for the amount of capital required to serving depreciation, $\delta k$, and after providing the existing amount of capital per worker to net new workers joining the labor force, $nk$
Steady State

- Assume that the productivity of labor (A) does not change.
- In such a case, there will be a state in which the capital per worker and the output per worker is not changing – i.e. $\Delta k = 0$.
- That’s the steady state.
- That is, $\Delta k = sf(k) - (\delta + n)k = 0$.
- Or, $sf(k^*) = (\delta + n)k^*$.
- Level of capital per worker at the steady state is $k^*$.
The curves intersect at point A, the "steady state". At the steady state, output per worker is constant. However, total output is growing at the rate of n, the rate of population growth.
An increase in the saving rate shifts the function up. Saving per worker is now greater, so capital accumulation increases, shifting the steady state from point A to B. Income goes from $y_0$ to $y_1$. Initially the economy expands faster, but eventually goes back to the steady state rate of growth which equals $n$. There is now permanently higher capital and productivity per worker, but economic growth is the same as before the savings increase.
Solow Growth Model (3)
Short-run Implications

- Policy measures like tax cuts or investment subsidies can affect the steady state level of output but not the long-run growth rate.
- Growth is affected only in the short-run as the economy converges to the new steady state output level.
- The rate of growth as the economy converges to the steady state is determined by the rate of capital accumulation.
- Capital accumulation is in turn determined by the savings rate (the proportion of output used to create more capital rather than being consumed) and the rate of capital depreciation.
Long-run Implications

• The long-run rate of growth is exogenously – determined outside of the model

• Predicts that an economy will always converge towards a steady state rate of growth, which depends only on the rate of technological progress and the rate of labor force growth.

• A country with a higher saving rate will experience faster growth, e.g. Singapore had a 40% saving rate in the period 1960 to 1996 and annual GDP growth of 5-6%, compared with Kenya in the same time period which had a 15% saving rate and annual GDP growth of just 1%.